Solution by ANNA L. VAN BEUSCHOTEN, Professor of Mathematics, Wells College, Aurora, N. Y.

Let the three straight lines be given by the equation

$$y=ax+b,\ y=cx+d,\ y=ex+f.$$

The condition that these lines intersect in a common point is given by the vanishing of the determinant,

$$\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{vmatrix}$$

But the vanishing of this determinant is also the condition that the points $(a,\ b)$, $(c,\ d)$, $(e,\ f)$ are colinear.

Also solved by *G. B. M. ZERR*.

177. Proposed by GEORGE LILLEY, Ph. D., LL. D., University of Oregon, Eugene, Ore.

If two medians of a triangle intersect each other at right angles, the third median will be the hypotenuse of a right triangle, of which the other two will be the sides.

Solution by H. B. PENHOLLOW, DeWitt Clinton High School, New York, N. Y.

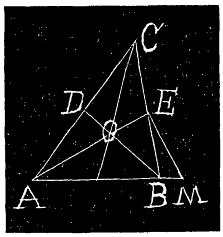
Given $\triangle ABC$, medians meeting at O, having $\angle AOB$ a right angle.

From E draw EM perpendicular to AE, meeting AB produced in M. Then $\triangle AEM$ is a right triangle in which AE is one median, $EM=DB$ another median. Also since triangles AEM and AOB are similar, $AE/AO=AM/AB$. But $AE=\frac{3}{2}AO$.

$\therefore\ AM=\frac{3}{2}AB$.

Also OF is median of right triangle AOB.

$\therefore\ OF=\frac{1}{2}AB$, or $CF=\frac{3}{2}AB=AM$. Q. E. D.



Also solved by *P. S. BERG*, *HENRY HEATON*, *P. H. PHILBRICK*, *C. A. LINDEMANN*, *G. I. HOPKINS*, *S. E. HARWOOD*, *J. F. LAWRENCE*, *T. T. DAVIS*, *G. B. M. ZERR*, and *ANNA BENCHOTEN*. Professor Penhollow and Miss Benchoten each furnished three solutions.

178. Proposed by JOHN M. ARNOLD, Crompton, R. I.

A cylinder thirty feet long and two feet in diameter is to be placed in a machinery car, the inside dimensions of which are eight feet wide and eight feet high. Find length of the shortest car that will contain it.

No correct solution of this problem has been received.

179. Proposed by ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi, University of Mississippi.

Of all isosceles triangles inscribed in a circle, the equilateral is the maximum and has the maximum perimeter. Prove geometrically.

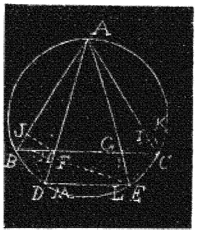
Solution by the PROPOSER.

Case I. (See Fig. 1.) Vertical angle of isosceles triangle less than 60°.

Let ABC be an inscribed equilateral triangle and ADE any inscribed isosceles triangle with its base DE parallel to BC.

1st. To prove $\triangle ABC$ greater than $\triangle ADE$. $\triangle AFG$ is common.

Through D draw a parallel to EA meeting BC at H, and forming the parallelogram $DHGE$. It is easily proved that H always lies to the right of B.



Hence, HG is less than BG. Draw the diagonal HE.

Comparing triangles HGE and AGC,

$$\triangle HGE : \triangle AGC = HG.GE : AG.GC.$$

In $\triangle BAG$, BG is less than AG, and therefore, HG is less than AG. Also, in $\triangle GEC$, GE is less than GC.

Therefore, $\triangle HGE$ is less than $\triangle AGC$.

Hence parallelogram $HGED$ is less than $\triangle AGC + \triangle AFB$, and trapezoid $DFGE$, being a part of this parallelogram, is less than the sum of the same two triangles.

Therefore $\triangle ABC$ is greater than $\triangle ADE$.

2nd. To prove the perimeter of $\triangle ABC$ greater than that of $\triangle ADE$.

Draw FJ and GI perpendicular to AF and AG, respectively.

Draw CK perpendicular to GI produced, and FM and GL perpendicular to DE. AI is greater than AG, AJ is greater than AF, $FG = ML$.

In similar triangles CGK and GLE, CG is greater than EG, CK (and, therefore, CI) is greater than EL.

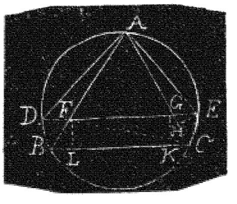
Similarly, BF is greater than DF, and BJ is greater than DM.

It follows that the perimeter of $\triangle ABC$ is greater than that of $\triangle ADE$.

Case II. (See Fig. 2.) Vertical angle of isosceles triangle greater than 60°.

Let ABC be an inscribed equilateral triangle and ADE an inscribed isosceles triangle with its base, DE, parallel to BC.

1st. To prove $\triangle ABC$ greater than $\triangle ADE$.



$\triangle AFG$ is common. Draw FC.

$\triangle FGC$ is greater than $\triangle AGE$, since $FG = AG$, and in $\triangle GCE$, GC is greater than GE.

It follows that trapezoid $BFGC$ is greater than the sum of triangles AGE and AFD.

Hence $\triangle ABC$ is greater than $\triangle ADE$.

2nd. To prove perimeter of $\triangle ABC$ greater than that of $\triangle ADE$.

On AC lay off AH equal to AE. Connect H and E.

$\angle AEH = \angle AHE$. $\angle GEH = \angle AEH - \angle AEG$.

Also, $\angle CEH = \angle AHE - \angle HCE$. But $\angle HCE = \angle AEG$.

Therefore, $\angle GEH = \angle CEH$, or HE is the bisector of $\angle GEC$.

Therefore $CH : HG = CE : GE$.

But CE is greater than GE.

Therefore CH is greater than HG, or CH is greater than one-half of CG.

Draw GK and FL perpendicular to BC. $KC = \frac{1}{2} GC$.

Adding, $KC + CH$ is greater than CG. But GC is greater than GE.

Therefore $KC + CH$ is greater than GE.

Hence $AC + CK$ is greater than $AE + EG$.

Similarly, $AB+BL$ is greater than $AD+DF$. And, since $LK=FG$, the perimeter of $\triangle ABC$ is greater than the perimeter of $\triangle ADE$.

Excellent demonstrations were received from *P. H. PHILBRICK*, *G. B. M. ZERR*, *HENRY HEATON*, *C. A. LINDEMAN*, and *T. T. DAVIS*.

180. Proposed by R. TUCKER, M. A.

ABC is a triangle; A', B', C' are the images of A, B, C with respect to BC, CA, AB. The circum-circle ABC cuts $A'BC$ (say) in K (on $A'B$), M (on $A'C$) and AK, AM, AA' cut BC in P, R, Q, respectively. Prove that (1) the orthocenters of the associated triangles lie on circle ABC; (2) triangle AKM has its sides parallel to and equal twice the sides of the pedal triangle of ABC, and is also equal triangle formed by the above-named orthocenters; (3) $CP.a=b^2$, $BR.a=c^2$, $AP.a=AR.a=bc$, $BP.a=a^2-b^2$, $CR.a=a^2-c^2$, *i. e.*, $PR.a=2bc\cos A$, (4) hence BA touches circle ARC, which contains a Brocard-point of ABC; similarly for CA and circle APB; (5) $BR.CR'$, $AR''=abc=CP.BP'$, AP'' (where R', R'', P', P'' correspond to R, P, on CA, AB. respectively); K, K' are the Brocard constants ($k=a^2+b^2+c^2$) of ABB, $A'B'C'$; then $K'=K=\triangle^2/R^2$.

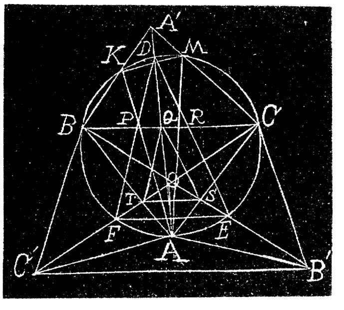
Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

(1) Since the triangles $A'BC$, $B'AC$, $C'AB$ are equal to ABC, respectively, and A', B', C' are the images of A, B, C, the orthocenters of the triangles are the images of the orthocenter O, of the triangle ABC with respect to its sides.

But $BS.SE=AS.SC$, or $a\sin C.SE=a\cos C.c\cos A$. $\therefore$ $SE=c\cos A\cot C=SO$.

Similarly, $DQ=b\cos C\cot B=QO$, $TF=a\cos B\cot A=TO$.

$\therefore$ D, E, F are the orthocentres of the triangles.



(2) ArcKDC=arcCEA, both measured by $\angle B$.

ArcDC=arcCE, both measured by $\angle(\frac{1}{2}\pi-C)$.

$\therefore$ ArcKD=arcAE and DE is parallel to KA.

ArcMKB=arcBFA, both measured by $\angle C$.

ArcDB=arcBF, both measured by $\angle(\frac{1}{2}\pi-B)$.

$\therefore$ ArcDM=arcFA, and DF is parallel to AM.

$\therefore$ ArcKBA=arcDBF, arcDCE=arcMEA, arcKDM=arcFAE.

$\therefore$ $DF=KA=2QT$, $DE=MA=2QS$, $KM=FE=2TS$.

Also DE is parallel to AK is parallel to QS, DF is parallel to MA is parallel to QT, FE is parallel to TS. $\triangle DFE=\triangle AKM$ (three sides of one equal three sides of other).

(3) From triangle PAC, $\angle PAC=\angle B$, $\angle P=\angle A$.

$\therefore$ $CP\sin A=b\sin B$ or $CP.a=b^2$.

Similarly, from triangle BAR, $BR\sin A=c\sin C$, or $BR.a=c^2$, $\angle P=\angle R=\angle A$.